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The origin of the radiatively induced Lorentz and CPT violations, in perturbative evaluations, of an extended version of QED, is investigated. Using a very general calculational method, concerning the manipulations and calculations involving divergent amplitudes, we clearly identify the possible sources of contributions for the violating terms. We show that consistency in the perturbative calculations, in a broader sense, leaves no room for the existence of radiatively induced contributions which is in accordance with what was previously conjectured and recently advocated by some authors supported on general arguments.

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The implications of Lorentz and CPT symmetry breaking have received a great deal of attention in the last few years. Most of them were all dedicated to the QED extended sector of the Extended Standard Model constructed by Colladay and Kostelecky [1]. They have developed a conceptual framework and a procedure for the treatment of spontaneous CPT and Lorentz violation within a context where the gauge structure and renormalizability are maintained [2]. The Lagrangian of the QED extended sector is composed by the usual QED theory in addition to the breaking terms

$$L^{SB} = -a_\mu \bar{\Psi}\gamma^\mu\Psi - b_\mu\bar{\Psi}\gamma_5\gamma^\mu\Psi + \frac{1}{2}k^\alpha\epsilon_{\alpha\lambda\mu\nu}A^\lambda F^{\mu\nu}. \tag{1}$$

In the above expression, a_μ and b_μ are real and constant prescribed four-vectors. The coupling k_α is also real and it has dimension of mass. The matrix γ_5 is the usual Hermitian Dirac matrix which is related to the totally antisymmetric tensor $\varepsilon_{\mu\nu\alpha\beta}$ through $tr\gamma_5\gamma_\alpha\gamma_\beta\gamma_\mu\gamma_\nu = 4i\epsilon_{\alpha\beta\mu\nu}$. The mathematical structure of the purely photonic sector breaking term, in the above Lagrangian, allows us to identify an important consequence for the modified QED theory. Due to the fact that it changes by a total derivative under potential gauge transformation ($A_\mu \rightarrow A_\mu + \partial_\mu\Lambda$), the action is not modified and the resulting equations of motions remain the same ones as those of the original theory. Such behavior is precisely what we call the Chern-Simons(CS) form. In spite of this, there are many phenomenological consequences associated to the modified theory [1] - [4]. However, all the present experimental and theoretical investigations seem to state that the k_μ coupling value, compatible with the phenomenology, is the identically zero one. Such statement does not completely eliminate the possibility of Lorentz and CPT breaking effects being present in the modified theory. Even that the k_μ coupling vanishes at the tree level, such type of effects can be, in principle, induced by radiative contributions. In the QED extended theory, radiative corrections coming from the fermionic sector can induce contributions of the CS form [2]. It could appear when the photon propagator is corrected by the b-breaking term. From the calculational point of view, we have to evaluate the usual QED one-loop vacuum polarization tensor with the free spin-1/2 fermion propagator, obeying the Dirac equation, changed by the inclusion of the b_μ coupling:

$$G(k) = \frac{i}{\not{k} - m - \not{b}\gamma_5}. \tag{2}$$

The corresponding one-loop amplitude can be written as

$$\Pi^{\mu\nu}(p) = \int \frac{d^4k}{(2\pi)^4} tr\left\{\gamma^\mu G(k)\gamma^\nu G(k+p)\right\}. \tag{3}$$

The evaluation of the above amplitude has been performed by many authors and different aspects involved in the calculations were emphasized [2] [5] - [8] [4] [9] - [12]. For a first set of authors and works, the obtained result is nonzero but it is essentially ambiguous. As a consequence, a definite value can be only stated after an arbitrary set of choices is taken. So, the results presented by the referred authors represent only a particular choice for the involved ambiguities. More recently, a second set of authors have argued, by using very general arguments, that only the definite zero value for the radiatively induced CS term is reasonable. Among them, G. Bonneau [9] shows that, if the theory is correctly defined by taking into account their Ward identities and appropriated normalization conditions, the CS term is absent in a non-ambiguous way. On the other hand, C. Adam and F.R. Klinkhamer [10], by requiring causality in addition to the validity of the perturbation theory, concluded that no CS term must exist. The same result was previously conjectured by Colladay and Kostelecky [2] and advocated by Coleman and Glashow [4]. So, it remains a question: Why do general physical arguments lead to a definite zero value while the perturbative calculations do not? On the other hand, given the fact that a nonzero Chern-Simons term represents Lorentz and CPT violation, we can ask which are the steps in the perturbative evaluation that give raise to the non-zero contribution. Talking in different words, which are the assumptions, taken in the intermediate steps, where resides the origin of the Lorentz and CPT violation in the perturbative calculation, that a consistent handling of divergences may avoid in order to obtain the expected value for the CS term, the definite zero value?

The main point is the following: If the CS term is non-vanishing, it represents a Lorentz and CPT symmetry breaking induced by radiative corrections in the QED extended theory. However, in order to consider such phenomenon as a fundamental one, it must not be a simple consequence of a choice for arbitrariness, but it should emerge as an unavoidable aspect of the calculations. In other words, to define a calculational scheme such that some symmetries are violated in the calculations is a very simple job, but this does not mean that the corresponding phenomenon must exist. We have to put this particular calculation in accordance with other ones of the perturbative calculations, specially those which are necessary for the construction of the (symmetric) renormalizable Standard Model. This means to treat all the involved mathematical structures in the same way they are treated in the symmetric theory.

The purpose of the present work is precisely to clarify these points. We will use a very general calculational strategy to handle divergences [13] in order to isolate in a very clear way the possible contributions for the CS term in the perturbative evaluation, and show that when the interpretation required by the consistency in perturbative calculations is adopted, in a broader sense, an exactly zero value for the Lorentz and CPT contribution is achieved.

In order to evaluate the CS term we have to consider γ_5-odd divergent structures and therefore the Dimensional Regularization (DR) technique [14] is excluded from the possible tools. To make clear this point we write in the expression (3) the exact propagators $G(k)$, given in the expression (2), in the form [5]

$$G(l) = S(l) + G_b(l), \tag{4}$$

with

$$G_b(l) = \frac{1}{\not{k} - m - \not{b}\gamma_5}\ \not{b}\gamma_5 S(l), \tag{5}$$

and $S(l)$ being the usual spin-1/2 fermion propagator. After the substitution of the above expression, the $\Pi_{\mu\nu}(p)$ amplitude can be split in three terms: $\Pi^{\mu\nu} = \Pi_0^{\mu\nu} + \Pi_b^{\mu\nu} + \Pi_{bb}^{\mu\nu}$. The first contribution, $\Pi_0^{\mu\nu}$, is precisely the pure QED vacuum polarization tensor. The linear b-term is given by

$$\Pi_b^{\mu\nu}(p) = \int \frac{d^4k}{(2\pi)^4} tr\left\{\gamma^\mu S(l)\gamma^\nu G_b(l+p) + \gamma^\mu G_b(l+p)\gamma^\nu S(l)\right\}. \tag{6}$$

To evaluate $\Pi_b^{\mu\nu}$ to the lowest order in b, we simply replace the expression (5) by

$$G_b(k) = -iS(k)\ \not{b}\gamma_5 S(k). \tag{7}$$

Now, the corresponding expression to $\Pi_b^{\mu\nu}$ may be written as

$$\Pi_b^{\mu\nu}(p) \simeq b_\lambda \Pi^{\mu\nu\lambda}(p), \tag{8}$$

where

$$\begin{aligned}\Pi^{\mu\nu\lambda}(p) = (-i)\int \frac{d^4k}{(2\pi)^4} tr\left\{\gamma^\mu S(k)\gamma^\nu S(k+p)\gamma^\lambda\gamma_5 S(k+p) + \right.\\ \left. + \gamma^\mu S(k)\gamma^\lambda\gamma_5 S(k)\gamma^\nu S(k+p)\right\}.\end{aligned} \tag{9}$$

So, the crucial mathematical structure, which we need to evaluate in order to get the value for the CS term, is an AVV triangle amplitude. Such three-point function is a Green's function of the symmetric theory (the renormalizable Standard Model). There are many kinds of arbitrariness involved in the evaluation of such amplitude. The requirement of consistency in the perturbative calculation implies that the choices for the arbitrariness present in the above expression must be taken in a consistent way with those adopted in the construction of the symmetric Standard Model. Given this argument we will consider the most general mathematical expression and only after all the considerations relative to the consistency in the evaluation of perturbative amplitudes have been made, we will return to the specific situation of the eq.(9).

We start by the definition

$$T_{\lambda\mu\nu}^{AVV}=\int\frac{d^4k}{(2\pi)^4}tr\left\{\gamma_\mu\left[(\not k+\not k_1)-m\right]^{-1}\gamma_\nu\left[(\not k+\not k_2)-m\right]^{-1}i\gamma_\lambda\gamma_5\left[(\not k+\not k_3)-m\right]^{-1}\right\}, \tag{10}$$

which corresponds to the most general expression for the direct diagram. In the above expression k_1, k_2 and k_3 stand for arbitrary choices for the internal lines momenta. They are related to the external ones by their differences which we adopt: $k_3-k_1=p, k_1-k_2=p'$ and $k_3-k_2=p'+p=q$. Their summations are undefined physical quantities. If we are worried about consistency, in the evaluation of this superficially linearly divergent structure, the first step is the identification of eventual constraints that this amplitude should obey, in spite of its divergent character. Such constraints are invariably materialized through relations among other amplitudes and/or by fixing a kinematical limit through a low-energy theorem. Due to the fact that, in principle, all the Green's functions of the perturbative expansion are in some way related, we can use eventual constraints imposed by general physical grounds upon a particular amplitude to restrict other ones. For the AVV we note, for example, the identity

$$\begin{aligned}(k_3-k_2)_\lambda&\left\{\gamma_\nu\frac{1}{(\not k+\not k_2)-m}i\gamma_\lambda\gamma_5\frac{1}{(\not k+\not k_3)-m}\gamma_\mu\frac{1}{(\not k+\not k_1)-m}\right\}=-\left\{i\gamma_\nu\gamma_5\frac{1}{(\not k+\not k_3)-m}\gamma_\mu\frac{1}{(\not k+\not k_1)-m}\right\}\\ &-2mi\left\{\gamma_\nu\frac{1}{(\not k+\not k_2)-m}\gamma_5\frac{1}{(\not k+\not k_3)-m}\gamma_\mu\frac{1}{(\not k+\not k_1)-m}\right\}+\left\{\gamma_\nu\frac{1}{(\not k+\not k_2)-m}i\gamma_\mu\gamma_5\frac{1}{(\not k+\not k_1)-m}\right\}.\end{aligned} \tag{11}$$

The above identity, which has nothing to do with divergences, can be converted to a relation among perturbative Green's functions if we take the traces operation in both sides and next integrate in the momentum k. This gives us

$$(k_3-k_2)_\lambda T_{\lambda\mu\nu}^{AVV}(k_1,k_2,k_3;m)=-2miT_{\mu\nu}^{PVV}(k_1,k_2,k_3;m)+T_{\mu\nu}^{AV}(k_1,k_2;m)-T_{\nu\mu}^{AV}(k_3,k_1;m), \tag{12}$$

where we have introduced the PVV three-point function defined by

$$T_{\mu\nu}^{PVV}(k_1,k_2,k_3;m)=\int\frac{d^4k}{(2\pi)^4}Tr\left\{\gamma_5\frac{1}{(\not k+\not k_3)-m}\gamma_\mu\frac{1}{(\not k+\not k_1)-m}\gamma_\nu\frac{1}{(\not k+\not k_2)-m}\right\}, \tag{13}$$

and the AV two-point function

$$T_{\mu\nu}^{AV}(k_1,k_2;m)=\int\frac{d^4k}{(2\pi)^4}Tr\left\{i\gamma_\mu\gamma_5\frac{1}{(\not k+\not k_1)-m}\gamma_\nu\frac{1}{(\not k+\not k_2)-m}\right\}. \tag{14}$$

Following a similar procedure, two other relations can be produced by contracting the term between curly brackets on the left hand side of the eq.(11) with the external momenta $(k_3-k_1)_\mu$ and $(k_1-k_2)_\nu$. They are

$$\bullet(k_3-k_1)_\mu T_{\lambda\mu\nu}^{AVV}=T_{\lambda\nu}^{AV}(k_1,k_2;m)-T_{\lambda\nu}^{AV}(k_3,k_2;m) \tag{15}$$

$$\bullet(k_1-k_2)_\nu T_{\lambda\mu\nu}^{AVV}=T_{\lambda\mu}^{AV}(k_3,k_2;m)-T_{\lambda\mu}^{AV}(k_3,k_1;m). \tag{16}$$

The AV structure that appeared on the right hand side of the eqs.(12), (15) and (16), is a two-point function physical amplitude and possesses its own relations with other physical amplitudes. The most important one for our present purposes is the following

$$T_{\mu\nu}^{AV}(k_1,k_2;m)=-\frac{1}{2m}\varepsilon_{\mu\nu\alpha\beta}(k_1-k_2)_\alpha T_\beta^{SV}(k_1,k_2;m), \tag{17}$$

where we have introduced the SV two-point function defined as

$$T_\beta^{SV}(k_1,k_2;m)=\int\frac{d^4k}{(2\pi)^4}Tr\left\{\hat{1}\frac{1}{(\not k+\not k_1)-m}\gamma_\beta\frac{1}{(\not k+\not k_2)-m}\right\}. \tag{18}$$

The eq.(17) can be stated before the introduction of the integration sign.

At this point we can ask ourselves for the meaning of the eqs.(12), (15), (16) and (17) and why they are important for our present investigation. First, we note that all the involved mathematical structures are, in principle, divergent quantities. This means that, in order to specify in a definite way the corresponding physical amplitudes, it will be necessary to handle undefined mathematical quantities. This implies in taking choices for the arbitrariness involved. Since we cannot run away from some assumptions, the unique guides we have at our disposal are the physical constraints we can eventually identify. The eqs.(12), (15) and (16) work like constraints for the explicit calculations, i.e., when we evaluate the AVV amplitude and after this contract the obtained expression, it must be possible to identify mathematical structures identical to those obtained in the evaluation of the AV and PVV functions previously calculated by the same methods. The importance of the identities resides in the fact that through such relations we can submit the decisions about the involved arbitrariness, in the evaluation of the AVV amplitude, from which the CS term should be extracted, to the physical constraints imposed to the AV and SV two-point functions. Due to the eqs.(12), (15) and (16), such structures are expected to be identified in the evaluation of the AVV amplitude. This aspect is crucial for the controversy about the value for the Chern-Simons term in the Extended QED. In order to show what we have announced, we first note that if we evaluate the traces involved in the AVV structure, the answer can be written in the form [13] [11]

$$t_{\lambda\mu\nu}^{AVV} = -4\left\{-f_{\lambda\mu\nu} + n_{\lambda\mu\nu} + m_{\lambda\mu\nu} + p_{\lambda\mu\nu}\right\}, \tag{19}$$

where, after the integration, only $n_{\lambda\mu\nu}$ will acquire a linear divergence's degree. It is explicitly given by

$$n_{\lambda\mu\nu} = \varepsilon_{\mu\nu\lambda\alpha}\frac{(k+k_2)\cdot(k+k_3)(k+k_1)_\alpha}{\left[(k+k_1)^2-m^2\right]\left[(k+k_2)^2-m^2\right]\left[(k+k_3)^2-m^2\right]}, \tag{20}$$

which can be conveniently reorganized as

$$\begin{aligned} n_{\lambda\mu\nu} = {} & \frac{\varepsilon_{\mu\nu\lambda\alpha}}{4}\left\{\frac{2k_\alpha+(k_1+k_2)_\alpha}{\left[(k+k_1)^2-m^2\right]\left[(k+k_2)^2-m^2\right]} + \frac{2k_\alpha+(k_1+k_3)_\alpha}{\left[(k+k_1)^2-m^2\right]\left[(k+k_3)^2-m^2\right]}\right\} \\ & + \frac{\varepsilon_{\mu\nu\lambda\alpha}}{4}\left\{\frac{(k_1-k_2)_\alpha}{\left[(k+k_1)^2-m^2\right]\left[(k+k_2)^2-m^2\right]} + \frac{(k_1-k_3)_\alpha}{\left[(k+k_1)^2-m^2\right]\left[(k+k_3)^2-m^2\right]}\right. \\ & \left. + \left[2m^2-(k_2-k_3)^2\right]\frac{2(k+k_1)_\alpha}{\left[(k+k_1)^2-m^2\right]\left[(k+k_2)^2-m^2\right]\left[(k+k_3)^2-m^2\right]}\right\}. \end{aligned} \tag{21}$$

The first two terms contain now all the linear divergence and the ambiguous combination of the arbitrary internal lines momenta. Given the identity (17) it is expected that such terms are related to SV two-point functions. In fact, it is easy to verify that

$$4m\frac{2k_\alpha+(k_i+k_j)_\alpha}{\left[(k+k_i)^2-m^2\right]\left[(k+k_j)^2-m^2\right]} = tr\left[\hat{1}\frac{1}{(\not{k}+\not{k}_i)-m}\gamma_\alpha\frac{1}{(\not{k}+\not{k}_j)-m}\right]. \tag{22}$$

After the integration in the momentum k, the right hand side can be identified with the SV two-point function defined in eq.(18). The important aspect involved resides in the fact that all the undefined pieces present in the AVV amplitude are linked with the value of the SV physical amplitude. Consequently, we can make use of the eventual physical constraints, to be imposed on the SV amplitude, to guide us in taking the consistent choices for the corresponding arbitrariness present in the AVV amplitude.

After these important remarks, in order to give additional steps to our investigations, some manipulations and calculations involving divergent amplitudes are required. This means to specify some strategy to handle the problem. We adopt the calculational strategy introduced by one of us [13] [16] [15] [11] in order to specify the Feynman integrals which are necessary for the evaluations of all the Green's functions involved in the present discussion.

First, we consider the two-point structures defined by

$$(I_2; I_2^\mu) = \int\frac{d^4k}{(2\pi)^4}\frac{(1; k^\mu)}{\left[(k+k_1)^2-m^2\right]\left[(k+k_2)^2-m^2\right]}, \tag{23}$$

which are given by

$$\bullet I_2 = I_{log}(m^2) - \left(\frac{i}{(4\pi)^2}\right)Z_0((k_1-k_2)^2; m^2) \tag{24}$$

$$\bullet\,(I_2)_\mu = -\frac{1}{2}(k_1+k_2)_\alpha\Delta_{\alpha\mu} - \frac{1}{2}(k_1+k_2)_\mu\,(I_2), \tag{25}$$

where we have introduced, in a more compact notation, the two-point function structures [13]

$$Z_k(\lambda_1^2,\lambda_2^2,q^2;\lambda^2)=\int_0^1 dzz^k ln\left(\frac{q^2 z(1-z)+\left(\lambda_1^2-\lambda_2^2\right)z-\lambda_1^2}{-\lambda^2}\right), \tag{26}$$

and the basic divergent objects

$$\bullet\Delta_{\mu\nu}=\int_\Lambda \frac{d^4k}{(2\pi)^4}\frac{4k_\mu k_\nu}{(k^2-m^2)^3}-\int_\Lambda \frac{d^4k}{(2\pi)^4}\frac{g_{\mu\nu}}{(k^2-m^2)^2} \tag{27}$$

$$\bullet I_{log}(m^2)=\int_\Lambda \frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-m^2)^2}. \tag{28}$$

According to the same prescription we can also calculate the integrals

$$(I_3;I_3^\mu;I_3^{\mu\nu})=\int \frac{d^4k}{(2\pi)^4}\frac{(1;k^\mu;k^\mu k^\nu)}{[(k+k_1)^2-m^2]\,[(k+k_2)^2-m^2]\,[(k+k_3)^2-m^2]}. \tag{29}$$

We write the results as

$$\bullet I_3=\left(\frac{i}{(4\pi)^2}\right)\xi_{00} \tag{30}$$

$$\bullet\left(I_3\right)_\mu=\left(\frac{i}{(4\pi)^2}\right)\left\{(k_1-k_2)_\mu\xi_{01}-(k_3-k_1)_\mu\xi_{10}\right\}-k_{1\mu}I_3 \tag{31}$$

$$\begin{aligned}\bullet\left(I_3\right)_{\mu\nu}=&\left(\frac{i}{(4\pi)^2}\right)\Big\{-\frac{g_{\mu\nu}}{2}\left[\eta_{00}\right]\ +(k_1-k_2)_\mu(k_1-k_2)_\nu\xi_{02}+(k_3-k_1)_\mu(k_3-k_1)_\nu\xi_{20}\\&-(k_1-k_2)_\mu(k_3-k_1)_\nu\xi_{11}-(k_1-k_2)_\nu(k_3-k_1)_\mu\xi_{11}\Big\}\\&+\frac{g_{\mu\nu}}{4}\left[I_{\log}\left(m^2\right)\right]+\frac{\Delta_{\mu\nu}}{4}-k_{1\mu}\left(I_3\right)_\nu-k_{1\nu}\left(I_3\right)_\mu+k_{1\nu}k_{1\mu}I_3.\end{aligned} \tag{32}$$

Here we have introduced the three-point function structures ξ_{nm} defined as

$$\xi_{nm}(k_1-k_2,k_3-k_1;m)=\int_0^1 dz\int_0^{1-z}dy\frac{z^n y^m}{Q(y,z)}, \tag{33}$$

where $Q(y,z)=(k_1-k_2)^2\,y(1-y)+(k_3-k_1)^2\,z(1-z)+2\,(k_1-k_2)\cdot(k_3-k_1)\,yz-m^2$, and

$$\eta_{00}=\frac{1}{2}Z_0((k_3-k_2)^2;m^2)-\left(\frac{1}{2}+m^2\xi_{00}\right)+\frac{1}{2}\left(k_3-k_1\right)^2\xi_{10}+\frac{1}{2}\left(k_1-k_2\right)^2\xi_{01}. \tag{34}$$

This systematization is sufficient for the present discussions. The main point is to avoid the explicit evaluation of such divergent structures, in which case a regulating distribution needs to be specified.

It is important, at this point, to emphasize the general aspects of the method. No shifts have been performed and, in fact, no divergent integrals have been calculated. All the final results produced by this approach can be mapped into those of any specific technique. The finite parts are the same as they should be by physical reasons. The divergent parts can be easily obtained . All we need is to evaluate the remaining divergent structures. By virtue of this general character, the present strategy can be simply used to systematize the procedures, even if one wants to use traditional techniques. Those parts that depend on the specific regularization method are naturally separated allowing us to analyze such dependence in a particular problem. Let us now use the above results to calculate the physical amplitudes.

Substituting the values for the Feynman integrals in the corresponding expressions for the AV and SV two-point functions, eq.(14) and eq.(18), we get

$$\bullet T_\mu^{VS}(k_1,k_2;m)=(-)4m(k_1+k_2)_\beta[\triangle_{\beta\mu}] \tag{35}$$

$$\bullet T_{\mu\nu}^{AV}(k_1,k_2;m)=2\varepsilon_{\mu\nu\alpha\beta}(k_2-k_1)_\beta(k_1+k_2)_\xi\triangle_{\xi\alpha}. \tag{36}$$

Note that the relation (17) is preserved by the performed calculations. We focus on the fact that, in spite of the potentially ambiguous character of the AV and SV functions, the identity (17) relating them is a non-ambiguous one. The above expressions are the most general ones for both mathematical structures. All the intrinsic arbitrariness are still present in the result. They are the undefined mathematical structure $\triangle_{\mu\nu}$ and the ambiguous combination of the internal momenta $k_i + k_j$. In order to give a definite result for the physical amplitudes, the arbitrariness should be removed through choices, which must be made preserving the physical requirements. So, we can ask if there are general aspects of QFT or symmetry determinations constraining the values for the SV and AV structures. In fact, it is easy to see that both two-point functions can be constrained to the identically zero value by general arguments. First, we note that, due to unitarity, if a two-point function like those considered is non-zero, it must have an imaginary part at the threshold $(k_1 - k_2)^2 = 4m^2$ (Cutkosky's rules). The remaining arbitrariness involved in the expressions (35) and (36) cannot introduce such content to the SV and AV two-point functions. So, if a non-zero value for the SV and AV amplitudes is assumed, as a consequence of some choices, the unitarity is violated in both cases. In addition, if a non-zero value is taken, the Lorentz invariance in the SV amplitude and the CPT symmetry in the AV amplitude are broken. A connection between a scalar and a vector particle is stated by the SV two-point function, and a connection between an axial and a vector particle through the AV two-point function. Another argument comes from the Ward identities analysis. The contraction with the external momentum $k_1 - k_2$ must lead us to a definite zero value for vector Lorentz indexes, and to a proportionality between the axial and the pseudo-scalar one in the case of the axial-vector Lorentz indexes. There is no consistent interpretation apart from the zero value for the AV amplitude since both contractions lead us to a vanishing value. In order to get the physically consistent result, we have at our disposal two options: to choose the internal lines momenta such that $k_1 + k_2 = 0$ or to select a regularization by the constraint $\triangle_{\mu\nu}^{reg} = 0$. We note that due to the identity (17) the values of the two-point functions SV and AV are intimately related. From the point of view of the Dimensional Regularization (DR), only the SV one can be treated and the result is identically vanishing. Due to the presence of the γ_5 Dirac matrix, or the totally antisymmetric tensor $\varepsilon_{\mu\nu\alpha\beta}$, such a treatment cannot be performed in the AV amplitude. The strategy we have adopted can be equally applied to both cases and shows us that it is not reasonable to make choices that lead us to a zero value for one of them and to a non-zero for the other one.

Given this argumentation we can immediately identify in these structures, which are contained in the AVV amplitude, the source of the Lorentz and CPT symmetry breaking in the evaluation of the CS term. Any choices for the ambiguities present in the AVV structure [18] [19], which imply in the attribution of a nonzero value for these contributions, will, in fact, generate Lorentz and CPT violations, because such choices produce non-vanishing AV and SV structures. So, the corresponding contributions for the CS term cannot be considered as an implication of the QED Extended theory but as a consequence of the adoption of an interpretation for the arbitrariness involved, which is clearly not consistent. The importance of this conclusion can be viewed if we evaluate $\Pi_{\lambda\mu\nu}(p)$ using the most general expression for the AVV mathematical structure [13] [17]

$$\Pi_{\lambda\mu\nu}(p) = \left(\frac{1}{2\pi^2}\right)\varepsilon_{\mu\nu\lambda\beta}p_\beta + 2i\left\{\varepsilon_{\mu\nu\beta\sigma}p_\beta\Delta_{\lambda\sigma} - \varepsilon_{\mu\nu\lambda\beta}p_\sigma\Delta_{\beta\sigma}\right\}. \tag{37}$$

In order to be consistent with the above discussion, we must choose $\triangle_{\mu\nu}^{reg} = 0$ eliminating then the contribution coming from the ambiguous terms. So, given this fact, can we conclude that the contribution for the CS term coming from $\Pi_{\lambda\mu\nu}(p)$ is the (non-ambiguous) value $\left(\frac{1}{2\pi^2}\right)\varepsilon_{\mu\nu\lambda\beta}p^\beta$? Not yet! We must show that the expression used to extract the above equation for $\Pi_{\lambda\mu\nu}(p)$ is in agreement with the symmetry content of the QED extended theory. In particular, the U(1) gauge symmetry was not assumed broken in the construction of the extended theory. Another important aspect is that concerning the low-energy limits. It is well-known that the AVV amplitude should obey the soft limit: $\lim_{q_\lambda \to 0} q^\lambda T_{\lambda\mu\nu}^{AVV} = 0$. The most minimum consistency requirement would force us to put any calculation of the AVV structure in accordance with these very general symmetry aspects. Otherwise, an eventual value for the CS term again could not be considered as a consequence of the extended theory but of the violation of other fundamental symmetries in the intermediate steps of the calculations. It is a simple matter to check that, by taking the explicit expression for the AVV amplitude and by contracting with the external momenta, we obtain [13]

$$\bullet\,(k_3 - k_1)_\mu T_{\lambda\mu\nu}^{AVV} = \left(\frac{i}{8\pi^2}\right)\varepsilon_{\nu\beta\lambda\xi}(k_1 - k_2)_\beta (k_3 - k_1)_\xi \tag{38}$$

$$\bullet\,(k_1 - k_2)_\nu T_{\lambda\mu\nu}^{AVV} = -\left(\frac{i}{8\pi^2}\right)\varepsilon_{\mu\beta\lambda\xi}(k_1 - k_2)_\beta (k_3 - k_1)_\xi \tag{39}$$

$$\bullet\,(k_3 - k_2)_\lambda T_{\lambda\mu\nu}^{AVV} = -\left(\frac{i}{4\pi^2}\right)\varepsilon_{\mu\nu\alpha\beta}(k_3 - k_1)_\alpha (k_1 - k_2)_\beta \left[2m^2\xi_{00}\right], \tag{40}$$

where $\xi_{00}((k_3 - k_2)^2 = 0) = \frac{-1}{2m^2}$. We can identify the expression on the right hand side of the eq.(40) as the calculated PVV amplitude. Both ingredients above mentioned are absent from the AVV amplitude. This is not surprising because it is the same situation we find in the Sutherland-Veltman paradox [20], connected with the pion decay phenomenology.

The above equation represents a manifestation of a fundamental phenomenon: the AVV triangle anomaly. Note that it involves a different type of arbitrariness in the perturbative calculations. It is not associated to the ones related to divergences aspects. The terms which violate the U(1) gauge symmetry come from finite contributions and therefore are not affected by an eventual regularization scheme. The AVV (symmetrized) physical amplitude must be constructed (in an arbitrary way) by subtracting the violating term

$$\left(T^{AVV}_{\lambda\mu\nu}\left((k_3-k_1),(k_1-k_2)\right)\right)_{phys} = T^{AVV}_{\lambda\mu\nu}\left((k_3-k_1),(k_1-k_2)\right) - T^{AVV}_{\lambda\mu\nu}(0), \tag{41}$$

where

$$T^{AVV}_{\lambda\mu\nu}(0) = -\left(\frac{i}{8\pi^2}\right)\varepsilon_{\mu\nu\lambda\beta}\left[(k_3-k_1)_\beta - (k_1-k_2)_\beta\right]. \tag{42}$$

The resulting amplitude preserves the U(1) gauge symmetry and it is in agreement with the low-energy theorem, in spite of violating the axial Ward identity involved. This procedure is precisely the one followed in the construction of the renormalizability of the Standard Model. So, again the consistency in the perturbative calculations requires the same interpretation for the same Green's function. This means to adopt for the AVV amplitude the expression

$$\begin{aligned}
\left(T^{AVV}_{\lambda\mu\nu}\right)_{Phys} = & \left(\frac{i}{(4\pi)^2}\right)(-4)(k_3-k_1)_\xi (k_2-k_1)_\beta \Big\{\varepsilon_{\nu\lambda\beta\xi}[(k_3-k_1)_\mu(\xi_{20}+\xi_{11}-\xi_{10}) \\
& \qquad\qquad + (k_2-k_1)_\mu(\xi_{11}+\xi_{02}-\xi_{01})] \\
& \qquad + \varepsilon_{\mu\lambda\beta\xi}[(k_3-k_1)_\nu(\xi_{11}+\xi_{20}-\xi_{10}) \\
& \qquad\qquad + (k_2-k_1)_\nu(\xi_{02}+\xi_{11}-\xi_{01})] \\
& \qquad + \varepsilon_{\mu\nu\beta\xi}[(k_3-k_1)_\lambda(\xi_{11}-\xi_{20}+\xi_{10}) \\
& \qquad\qquad - (k_2-k_1)_\lambda(\xi_{02}-\xi_{01}-\xi_{11})]\Big\} \\
& - \left(\frac{i}{(4\pi)^2}\right)\varepsilon_{\mu\nu\lambda\beta}(k_3-k_1)_\beta\Big\{Z_0\left((k_1-k_3)^2;m^2\right) - Z_0\left((k_2-k_3)^2;m^2\right) \\
& \qquad + \left[2(k_3-k_2)^2 - (k_1-k_3)^2\right]\xi_{10} + \\
& \qquad - (k_1-k_2)^2\xi_{01} + \left[1-2m^2\xi_{00}\right]\Big\} \\
& - \left(\frac{i}{(4\pi)^2}\right)\varepsilon_{\mu\nu\lambda\beta}(k_2-k_1)_\beta\Big\{Z_0\left((k_1-k_2)^2;m^2\right) - Z_0\left((k_2-k_3)^2;m^2\right) \\
& \qquad + \left[2(k_3-k_2)^2 - (k_1-k_2)^2\right]\xi_{01} \\
& \qquad - (k_3-k_1)^2\xi_{10} + \left[1-2m^2\xi_{00}\right]\Big\} - T^{AVV}_{\lambda\mu\nu}(0).
\end{aligned} \tag{43}$$

Now, taking the kinematical situation where the CS term is defined, eq.(9), we get

$$\Pi_{\mu\nu\lambda}(p) = \left(\frac{1}{2\pi^2}\right)\varepsilon_{\mu\nu\lambda\beta}p_\beta - iT^{A\to VV}_{\lambda\mu\nu}(0). \tag{44}$$

Identifying then $T^{AVV}_{\lambda\mu\nu}(0)$ as the violating term on the left hand side of the eq.(38) and (39) this means that the identically vanishing value is obtained.

So, a clean and sound conclusion is extracted: the consistency in perturbative calculations leave no room for the existence of the radiatively induced CS term in the extended QED. Therefore, if one wants to get a nonzero value for such contribution, it is necessary: **1)** to break in the intermediary steps of the calculation, Lorentz, CPT, unitarity and an axial Ward identity by attributing to mathematical structures, identical to the two-points functions AV and SV, which are related, a nonzero value or, **2)** to violate the low-energy theorem $\lim_{q_\lambda\to 0} q^\lambda T^{AVV}_{\lambda\mu\nu} = 0$, which may imply simultaneously in the violation of U(1) gauge symmetry in the Extended QED. The implication of the last sentence is the spoiling of the Standard Model renormalizability by destroying the anomaly cancellation mechanism. Any of such options clearly implies in ignoring the wider sense of the consistency in perturbative calculations, which means to treat the same Green's function in the same way in all places where they occur. If one does not consider these aspects, in fact, one can obtain Lorentz and CPT violation not only for the discussed problem but, following the same recipe, it is possible to state a copious number of similar situations in other theories and models.

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